

Revision 2

Year 11 Examination

Question/Answer Booklet

MATHEMATICS SPECIALIST UNITS 1 AND 2

Section Two:
Calculator-assumed

Student Number: In figures

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In words

Teacher name

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1

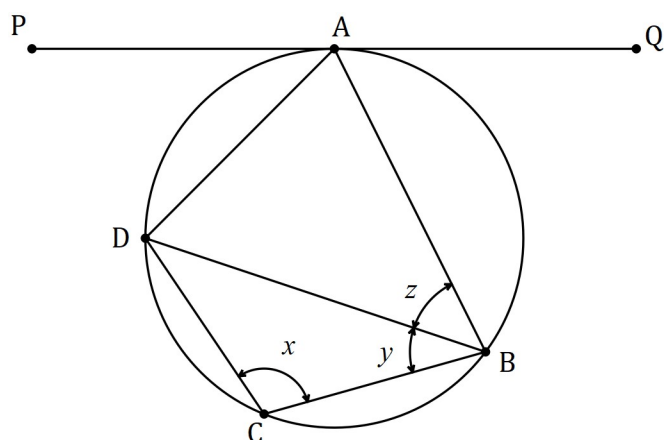
(6 marks)

- (a) The point $P(-3, 5)$ is translated by the column vectors $\begin{bmatrix} 2 \\ -7 \end{bmatrix}$ and $\begin{bmatrix} x \\ y \end{bmatrix}$ to $P'(9, -11)$. Determine the values of the constants x and y . (2 marks)
- (b) Determine the single matrix that represents, in order, the composition of a rotation of 30° anti-clockwise about the origin followed by reflection in the line $y = x$. Express matrix coefficients in exact form. (4 marks)

The diagram below shows four points A , B , C and D lying on the circumference of a circle. The line PQ is a tangent to the circle at A , $\angle BDC = 21^\circ$, $\angle PAD = 35^\circ$ and $\angle QAB = 62^\circ$.

Determine the size of angles x , y and z .

(3 marks)



(a) If $\mathbf{p} = 4\mathbf{i} - 2\mathbf{j}$ and $\mathbf{q} = 3\mathbf{i} + 2\mathbf{j}$ determine

(i) the angle between the directions of $\mathbf{p}$ and $\mathbf{q}$, to the nearest tenth of a degree.
(2 marks)

(ii) the scalar projection of $\mathbf{q}$ on $\mathbf{p}$.
(2 marks)

(b) The vector $21\mathbf{i} + 5a\mathbf{j}$ has a magnitude of 29 and is perpendicular to the vector $4\mathbf{i} - 2b\mathbf{j}$. Determine the values of the constants a and b , where $a < b$.
(5 marks)

(a) Prove that $\sin 3A = 3 \sin A - 4 \sin^3 A$.

(4 marks)

(b) Hence, or otherwise, solve $3 \sin A - 4 \sin^3 A = \frac{1}{2}$, $0 \leq A \leq \frac{\pi}{3}$.

(2 marks)

Triangle ABC has vertices $A(1, 2)$, $B(4, -1)$ and $C(5, 3)$.

- (a) The vertices ABC are transformed to $A'B'C'$ using matrix $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$. Write down the new coordinates of the vertices and describe the transformation. (4 marks)

- (b) The vertices ABC are transformed to $A''B''C''$ by matrix M so that the new coordinates of the vertices are $A''(-4, 3)$, $B''(2, 12)$ and $C''(-6, 15)$.

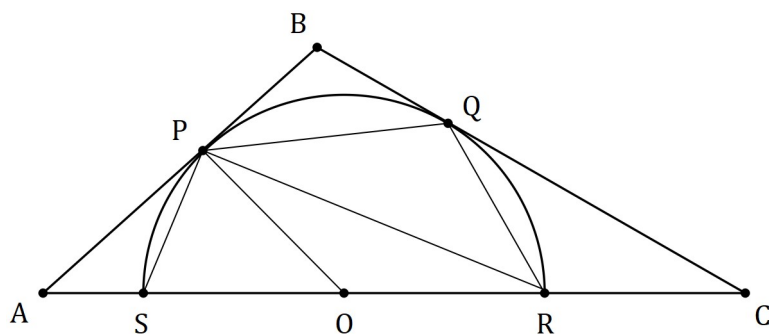
- (i) Determine the transformation matrix M . (3 marks)

- (ii) If the area of triangle ABC is k square units, express the area of triangle $A''B''C''$ in terms of k . (2 marks)

- (a) How many numbers must be chosen from the set of integers between 1 and 2017 inclusive to be certain that one of the numbers chosen is a multiple of 10. (3 marks)
- (b) A number is formed using four different digits chosen from those in the number 23 814. Determine how many different numbers can be formed that are
- (i) even. (1 mark)
- (ii) greater than 8 000. (1 mark)
- (iii) even or greater than 8 000. (2 marks)

The diagram shows a semi-circle, with diameter SR and centre O , circumscribed by triangle ABC , in which $\angle BAC = 42^\circ$ and $\angle BCA = 32^\circ$.

Determine, with reasons, the size of angles $\angle PRO$ and $\angle PRQ$.



Question 8**(8 marks)**

The sum of the first n terms of the sequence $2 + 8 + 14 + 20 + \cdots + (6n - 4)$ is $n(3n - 1)$.

(a) Show that this statement is true when $n = 5$. (2 marks)

(b) Use mathematical induction to prove the statement is true for $n \in \mathbb{Z}, n \geq 5$. (6 marks)